Embedding Factorizations for 3-uniform Hypergraphs

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In this talk, using the hypergraph amalgamation technique, two hypergraph embedding problems are solved.

1. The first solves the problem of embedding a hyperedge-colored copy of the complete 3-uniform hypergraph of order \( m \) into an \( r \)-factorization of the complete 3-uniform hypergraph of order \( n \), providing that \( n > 2m + \lceil (1 + \sqrt{8m^2 - 16m - 7})/2 \rceil \).

2. The second completely solves a related embedding problem, finding necessary and sufficient conditions for an embedding when not only are the colors of the hyperedges of the complete 3-uniform hypergraph of order \( m \) given, but also the colors of all the “pieces” of hyperedges on these \( m \) vertices are prescribed (the “pieces” of hyperedges are eventually extended to hyperedges of size 3 by adding new vertices during the embedding process).

Both these results make progress towards settling an old question of Cameron on completing partial 1-factorizations of hypergraphs.