On the Edge-Balance Index Sets of Envelope Graph of Stars, Paths and Cycles

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f$ of a graph $G$ is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$. An edge-friendly labeling $f : E(G) \to \mathbb{Z}_2$ induces a partial vertex labeling $f^+ : V(G) \to \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges incident on $x$ is 0 more than 1. Similarly, $f^+(x) = 1$ if the number of edges incident on $x$ is 1 more than 0. $f^+(x)$ is not define if the number of edges incident on $x$ is 1 equal to the number of edges labeled by 0. For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph $G$, $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| :$ the edge labeling $f$ is edge-friendly$\}$. Given a graph $G$, the envelope graph $EV(G)$ is the graph with $V(EV(G)) = V(G) \cup E(G)$ and $E(EV(G)) = E(G) \cup \{(u, (u, v)) : u \in V, (u, v) \in E(G)\}$. The edge-balance index sets of envelope graphs of stars, paths and cycles are presented.