Recursive Wavelength Routed Optical Network for Optical Network-on-Chips
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Abstract—The wavelength routed optical network (WRON) is a promising optical interconnection architecture that can be integrated into a System-on-Chip (SoC) with an intention to replace traditional wire-connected on-chip micro-networks which pose severe bandwidth limitations on future super large SoC chips. In this paper, we present a new recursive architecture, the Recursive Wavelength Routed Optical Network (RCWRON) based on the WRON, and provide the solution to the routing problem on RCWRON based on two of three routing parameters (the source node address, the destination node address and the routing wavelength). Compared with WRON, RCWRON has many advantages, such as its recursive structure and fault tolerance capability.

Keywords: Optical Network-on-Chips, wavelength, recursive, routing algorithm.

I. INTRODUCTION

As predicted by the International Technology Roadmap for Semiconductors (ITRS) [7], for the next 5 to 10 years, System-on-Chips (SoCs), using 45-nm (or 32 nm) transistors operating below one volt, will grow to multi-billion transistors running at a frequency of 10GHz or higher. One of the major challenges in designing such highly integrated SoCs is to find an efficient way to interconnect pre-designed Intellectual Property (IP) cores addressing both power and performance concerns. The continuously shrinking feature sizes, higher clock frequencies, and the simultaneous growth in complexity have made electrical interconnects a formidable task.

Optical interconnects offer the promise of decreasing interconnect delays and providing higher bandwidth to keep pace with transistor speed improvements [3][5], while potentially lowering power consumption and being resistant to Electromagnetic Interference (EMI) [6]. In specific, optical switches [1] and waveguides [2] can be used in Optical Network-on-Chip (ONoC) to realize the same function as a classical electronic router but with routing based on wavelength and with no need for an arbiter.

In [8], we generalized the Wavelength Routed Optical Network (WRON) [4] which is suitable to be used as the on-chip interconnection network in ONoC. However, the structure of WRON is not recursive, which makes it not scalable and causes difficulty in analyzing its property. In this paper, we propose a new recursive architecture, the Recursive Wavelength Routed Optical Network (RCWRON), to overcome the disadvantage of the WRON. The solution to the routing problem on RCWRON will also be provided.

The rest of the paper is organized as follows. Section I introduces the basic structure of the WRON and its routing scheme. In Section III, we present the Redundant Wavelength Routed Optical Network (RDWRON) as the basic building block to construct the RCWRON. Section IV presents the structure of RCWRON. Section V presents the solution to the routing problem on RCWRON. Section VI concludes the paper with suggestions for future exploration.

II. BASIC STRUCTURES OF ON-CHIP WAVELENGTH ROUTED OPTICAL NETWORK

The generalized WRON is composed of input/output nodes and multiple stages of optical (crossbar) switches. Each optical switch is associated with one resonant wavelength, \( w_r \). The operation of such an optical switch depends on the wavelength of the incoming signal to one of the input ports, \( w_i \). An incoming signal with \( w_i = w_r \) will pass through the switch on the same direction as the input signal (referred as the “straight” function); an incoming signal with \( w_i \neq w_r \) will pass through the switch on the cross direction (referred as the “across” function). Fig. 1 illustrates the two functions.

![Figure 1 Basic functions of an optical switch.](image)

In an \( N \)-WRON, where \( N \) represents the number of input/output nodes, the number of stages is equal to \( N \), except for the case when \( N=2 \). All the optical switches at the same stage share the same resonating wavelength. Fig. 2 illustrates the structure of a 4-WRON and a 5-WRON.

![Figure 2 Structure of a 4-WRON and a 5-WRON.](image)
In WRON, each routing path is associated with a three-element tuple \(<S, D, w>\), where \(S\) denotes the source node address, \(D\) denotes the destination node address, and \(w\) represents the assigned routing wavelength for data transmission. In general, for an \(N\)-WRON, given any two of the three parameters, the routing path is uniquely determined and the last parameter can be derived from the two known parameters. For an \(N\)-WRON, given two of the three parameters, \(S, D, w\), the other parameter can be derived according to the following equations \([8]\).

\[
D = f_D(N, S, w) =
\begin{cases}
1 - D^* & \text{if } D^* \leq 0 \\
D^* & \text{if } 0 < D^* \leq N \\
2 \times N + 1 - D^* & \text{if } D^* > N
\end{cases}
\quad (1)
\]

where \(D^* = S + (N - 2w + 1) \times (-1)^S\).

\[
S = f_S(N, D, w) =
\begin{cases}
1 - S^* & \text{if } S^* \leq 0 \\
S^* & \text{if } 0 < S^* \leq N \\
2 \times N + 1 - S^* & \text{if } S^* > N
\end{cases}
\quad (2)
\]

where \(S^* = D + (N - 2w + 1) \times (-1)^{N + D}\).

\[
w = f_w(N, S, D)
\]

When \(N\) is even, we have
\[
w = f_w(N, S, D) =
\begin{align*}
&\frac{N + 1 + S - D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d + 1 \\
&\frac{-N + S + D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d \quad \text{and } S + D > N \\
&\frac{N + S + D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d \quad \text{and } S + D \leq N \\
&\frac{N + 1 - S + D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d \\
&\frac{3N + 2 + S + D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d + 1 \quad \text{and } S + D \geq N + 2 \\
&\frac{N + 2 - S - D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d + 1 \quad \text{and } S + D < N + 2
\end{align*}
\]

and when \(N\) is odd, we have
\[
w = f_w(N, S, D) =
\begin{align*}
&\frac{N + 1 + S - D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d \\
&\frac{-N + S + D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d + 1 \quad \text{and } S + D > N \\
&\frac{N + S + D}{2} \quad \text{when } S = 2s \quad \text{and } D = 2d + 1 \quad \text{and } S + D \leq N \\
&\frac{N + 1 - S + D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d + 1 \\
&\frac{3N + 2 + S + D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d \quad \text{and } S + D \geq N + 2 \\
&\frac{N + 2 - S - D}{2} \quad \text{when } S = 2s + 1 \quad \text{and } D = 2d \quad \text{and } S + D < N + 2
\end{align*}
\]

III. 2-D REDUNDANT OPTICAL NETWORK

As shown in Section II, the WRON is capable of routing between any number of input and output nodes given enough wavelengths. However, the structure of WRON is not recursive. In another word, a WRON can not be built by combining smaller size WRONs. Another disadvantage of the non-recursive structure of WRON is the difficulty in studying the property of the network.

To solve this problem, in Section IV, we will propose a new recursive structure, 2-dimension RCWRON (2-D RCWRON), based on the WRON structure. In this section, we will first introduce the 2-dimension RDWRON (2-D RDWRON) as the basic building block to construct the 2-D RCWRON. The routing scheme of RDWRON will also be derived.

A. Inverse Connector (IC)

Inverse Connector (IC) and WRON are the basic units to construct the RDWRON. The function of an IC is to invert the source node address and the destination node address. We denote the IC with \(N\) source/destination nodes as \(N\)-IC with its structure shown in Fig. 3.

![Figure 3 Structure of an N-IC](image)

For an \(N\)-IC, if the source node address is \(S\), the destination node address \(D\) is \(D = N + 1 - S\).

B. Construction of 2-D RDWRON

A RDWRON with \(N\) input/output nodes is constructed by connecting \(N\)-WRONs and \(N\)-ICs alternatively as shown in Fig. 4. The resonant wavelengths in different stages in the RDWRON are preset as 1, 2, \(N^2\) from the first stage of the first \(N\)-WRON to the last stage of the last \(N\)-WRON in sequence.

![Figure 4 Structure of N^2-RDWRON](image)

We denote 2-D RDWRON with \(N\) input/output nodes as \(N^2\)-RDWRON. The structures of \(3^2\)-RDWRON and \(4^2\)-RDWRON are shown in Fig. 5(a) and Fig. 5(b), respectively.

Lemma 1. The total number of switches in an \(N^2\)-RDWRON is
\[
N \times \frac{N \times (N - 1)}{2} = \frac{N^2 \times (N - 1)}{2}.
\]
C. Features of 2-D RDWRON

The 2-D RDWRON has the following features:

- A set of different wavelengths can be used in routing to the same destination node from the same source node (by different routing paths).

- Different source nodes can use the same set of wavelengths to reach different destination nodes. By using these different wavelengths sets, all destination nodes can be reached from any source node.

**Lemma 2.** For an \(N^2\)-RDWRON with \(N\) inputs/outputs and \(N^2\) different wavelengths, all these \(N^2\) wavelengths can be partitioned into \(N\) different subsets \(\{W_1, W_2, \ldots, W_N\}\) where each subset \(W_i\) \((i=1, 2, \ldots, N)\) has exactly \(N\) different wavelengths such that the following three conditions are satisfied.

a) For each source node \(S_i\), any wavelength in the same subset \(W_i\) can lead to the same destination node.

b) For all source nodes, the partitions of \(N^2\) wavelengths into \(N\) subsets are same. When the partition is derived, it can be applied to all source nodes in which a) will be satisfied.

c) For each source node, different subset can be used to route to different destination nodes. Hence by using all \(N\) subsets, all \(N\) destination nodes can be reached from any source node.

The wavelength assignment of \(3^2\)-RDWRON and \(4^2\)-RDWRON are shown in Tab. 1 and Tab. 2, respectively.

**Table 1 Routing Wavelength Assignment of \(3^2\)-RDWRON.**

<table>
<thead>
<tr>
<th>(w)</th>
<th>(D_1)</th>
<th>(D_2)</th>
<th>(D_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1)</td>
<td>(w_2)</td>
<td>(w_3)</td>
<td>(w_4)</td>
</tr>
<tr>
<td>(S_2)</td>
<td>(w_5)</td>
<td>(w_6)</td>
<td>(w_7)</td>
</tr>
<tr>
<td>(S_3)</td>
<td>(w_8)</td>
<td>(w_9)</td>
<td>(w_1)</td>
</tr>
</tbody>
</table>

**Table 2 Routing Wavelengths of \(4^2\)-RDWRON.**

<table>
<thead>
<tr>
<th>(w)</th>
<th>(D_1)</th>
<th>(D_2)</th>
<th>(D_3)</th>
<th>(D_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1)</td>
<td>(w_1)</td>
<td>(w_2)</td>
<td>(w_3)</td>
<td>(w_4)</td>
</tr>
<tr>
<td>(S_2)</td>
<td>(w_5)</td>
<td>(w_6)</td>
<td>(w_7)</td>
<td>(w_8)</td>
</tr>
<tr>
<td>(S_3)</td>
<td>(w_9)</td>
<td>(w_{10})</td>
<td>(w_{11})</td>
<td>(w_{12})</td>
</tr>
<tr>
<td>(S_4)</td>
<td>(w_{13})</td>
<td>(w_{14})</td>
<td>(w_{15})</td>
<td>(w_{16})</td>
</tr>
</tbody>
</table>

It can be seen from Tab. 1 that all 9 (3\(^2\)) wavelengths are partitioned into three subsets \(W_1=\{1, 4, 7\}\), \(W_2=\{2, 5, 8\}\) and \(W_3=\{3, 6, 9\}\). This partition is same to all source nodes. For any source node, all wavelengths in the one subset will route to the same destination node. For instance, \(w_2, w_5\), and \(w_8\) in \(W_1\) can be used to route from \(S_1\) to \(D_1\). And any source node can use \(W_1\), \(W_2\) and \(W_3\) to reach all three destination nodes.

D. Routing Scheme of \(N^2\)-RDWRON

Similar to \(N\)-WRON, each routing path in \(N^2\)-RDWRON is associated with a three-element tuple \(<S, D, w>\). Given two of the three parameters \((S, D, w)\), the other parameter can be derived according to Propositions 1-3.

**Proposition 1.** For an \(N^2\)-RDWRON, given the source node address \(S\) and the routing wavelength \(w\), the destination node address \(D\) can be derived by

\[D = f_D(N, S, w_0),\]

where \(w_0 = \text{mod}(w-1, N)+1\), and \(f_D\) refers to the function given in Eqn. (1).

**Proposition 2.** For an \(N^2\)-RDWRON, given the destination node address \(D\) and the routing wavelength \(w\), the source node address \(S\) can be derived by

\[S = f_S(N, D, w_0),\]

where \(w_0 = \text{mod}(w-1, N)+1\), and \(f_S\) refers to the function given in Eqn. (2).

**Proposition 3.** In an \(N^2\)-RDWRON, a set of different routing wavelengths can be used in routing from one source node to one destination node. Denote the set of different wavelengths of the \(N^2\)-RDWRON as \(W\), given the RDWRON size \(N\), the source node address \(S\) and the destination node address \(D\), \(W\) can be derived by

\[W = \{w, w+N, w+2N, \ldots, w+(N-2)N, w+(N-1)N\},\]

where \(w = f_w(N, S, D)\), and \(f_w\) refers to the function defined in Eqn. (3).

The proof of Proposition 1-3 is shown in Appendix. Hence we have the following Lemma.

**Lemma 3.** For an \(N^2\)-RDWRON, given a set of \(N^2\) wavelengths \(W = \{w_j\} (j=1, 2, \ldots, N^2)\), \(W\) can be segmented into \(N\) subsets

\[W = \bigcup_{i=1, 2, \ldots, N} W_i\]

where \(W_i = \{w_{j1}, w_{j2+N1}, w_{j2+N1+N2}, \ldots, w_{j2+N1+N2+\cdots+N} \mid 1 \leq i \leq N\}\).

Given a source node \(S_i\), any path routed by the wavelengths in subset \(W_i (1 \leq i \leq N)\) will reach the same destination node \(D_i\), and vice versa.

E. Level2 RDWRON

The preset resonant wavelength for the RDWRON is not unique. In the following, we will introduce another type of RDWRON, named Level2 RDWRON, which will be used in the construction of RCWRON. Correspondingly, we name the previously introduced RDWRON which has the resonant wavelengths preset in sequence as Level1 RDWRON.

Let \(w_k\) be the wavelength preset at the \(k\)th stage in level2 RDWRON is, and \(w_k = i + (j - 1) \times N\),

\[
\begin{cases}
  j = \text{mod}(k-1, N)+1, \\
  i = \frac{k-1}{N}.
\end{cases}
\]

The following propositions can be derived for solving the routing scheme of the Level2 RDWRON.

**Proposition 4.** In a Level2 \(N^2\)-RDWRON, given the source
node address \( S \) and the routing wavelength \( w \), the destination node address \( D \) can be deriving as follows:
\[
D = f_D(N, S, w_0),
\]
where \( w_0 = \left\lfloor \frac{w-1}{N} \right\rfloor \).

**Proposition 5.** In a Level 2 \( N^2\)-RDWRON, given the destination node address \( D \) and the routing wavelength \( w \), the source node address \( S \) can be derived as follows:
\[
S = f_S(N, D, w_0),
\]
where \( w_0 = \left\lfloor \frac{w-1}{N} \right\rfloor \).

**Proposition 6.** In a Level 2 \( N^2\)-RDWRON, given the source node address \( S \) and the destination node address \( D \), the routing wavelength set \( W \) can be derived as follows:
\[
W = \{ (w-1)N+1, (w-1)N+2, \ldots, (w-1)N+(N-1), wN \},
\]
where \( w = f_w(N, S, D) \).

IV. STRUCTURE OF 2-D RECURSIVE OPTICAL NETWORK

A. Construction of 2-D RCWRON

As mentioned before, a 2-D RCWRON is constructed by RDWRONS. A 2-D RCWRON has two subnetworks, each composed of \( N \) \( N^2\)-RDWRONS. The RDWRONS in the first and second level are Level 1 RDWRONS and Level 2 RDWRONS, respectively. The wavelengths preset for RDWRONS in different level are different.

Each \( N^2\)-RDWRON in a 2-D RCWRON has \( N \) inputs/outputs and \( N^2 \) stages. Totally the \( N^2\)-RCWRON has \( N^2 \) inputs/outputs. Hence, we denote the RCWRON with \( N^2 \) inputs/outputs as \( N^2\)-RCWRON \((N>2)\). Fig. 6 shows the structure of an \( N^2\)-RCWRON \((N>2)\).

![Figure 6: Structure of 2-D \( N^2\)-RCWRON.](image)

The connection principle of \( N^2\)-RCWRON is explained as follows. The \( i^{th} \) output node of the \( j^{th} \) \( N^2\)-RDWRON is connected to the \( j^{th} \) input node of \( i^{th} \) \( N^2\)-RDWRON in the second level.

One may notice that the structure of an \( N^2\)-RCWRON is not unique. The basic rule is that each Level 1 RDWRON must have a connection to each Level 2 REWRON, and vice versa.

**Lemma 4.** The total number of switches in one 2-D \( N^2\)-RCWRON is
\[
O_{RCWRON} = 2 \times N \times \frac{N^2}{2} - \frac{2N^3}{2} - 1.
\]

B. Fault Tolerance Capability

Compared with the WRON, an outstanding advantage of RCWRON is its fault tolerance capability. As shown in Fig. 6, an \( N^2\)-RCWRON is composed of \( 2N \) RDWRONS, which are independent to each other. When one path fails, the faulty RDWRON can be easily identified by checking the sub-path in different levels of subnetworks. By abandoning the faulty RDWRON and the input/output nodes connected by the faulty RDWRON, the rest of the RCWRON can operate normally.

V. ROUTING SCHEME OF 2-D RCWRON

The key idea of the routing scheme for the 2-D RCWRON is to decompose the routing path into two parts, each corresponding to the sub-paths in two subnetworks, respectively, and then solve the routing problem in each RDWRON.

In the following, we will first present the rules of assigning routing wavelengths before we describe the routing scheme.

A. Routing Wavelength Assignment

According to Lemma 3, \( N^2 \) different wavelengths can be partitioned into two different subset groups \( W^{(1)} \) and \( W^{(2)} \). For all Level 1 RDWRONS, assign the wavelength subsets in \( W^{(1)} \) as their routing wavelengths. For all Level 2 RDWRONS, assign wavelength subsets in \( W^{(2)} \) as their routing wavelengths.

For any routing path \( P \) in RCWRON using wavelength \( w \), it can be decomposed into two parts, \( P_1 \) and \( P_2 \), where \( P_1 \) is the routing path in the first subnetwork (i.e., Level 1 RDWRONS) and \( P_2 \) is the routing path in the second subnetwork (i.e., Level 2 RDWRONS). For \( P_1 \), the routing wavelength \( w \) must be in one of the subsets in the Partition 2, denoted as \( W^{(2)} \). And for \( P_2 \), the routing wavelength \( w \) must be in one of the subsets in the Partition 1, denoted as \( W^{(1)} \). The following lemmas elaborate one way to determine such \( w \).

**Lemma 5.** Given a set \( W \) with \( N^2 \) different elements, there exist at least 2 different ways to partition \( W \) into \( N \) subsets:

\[
W = W^{(1)} = \bigcup_{m=1}^{N} W_m^{(1)}
\]

and

\[
W = W^{(2)} = \bigcup_{n=1}^{N} W_n^{(2)},
\]

where each subset has \( N \) elements such that for any two different subsets \( W_m^{(1)} \subset W^{(1)} \) and \( W_n^{(2)} \subset W^{(2)} \) \((m, n=1,2,\ldots,N)\), there is one and only one common element, i.e.,
Lemma 6. There are totally $N^2$ different combinations of $W_m^{(1)}$ and $W_n^{(2)}$, where $W_m^{(1)} \subset W_m$ and $W_n^{(2)} \subset W_n$ (m, n=1,2,...,N). All $N^2$ common elements for the $N^2$ combinations of $W_m^{(1)}$ and $W_n^{(2)}$ are different from each other and these common elements compose the $N^2$ different elements of set $W$.

Proof: Assume that $W = \{w_t \mid t = 1,2,...,N^2\}$, then an $N \times N$ matrix $M$ can be generated as 

$$M(i,j) = w_t, \quad \text{if } t = (i-1) \times N + j, \quad 1 \leq i,j \leq N,$$

where $i$ and $j$ denote the row and column number in $M$, respectively.

The two partitions of $W$ can be obtained as follows.

$$W = W^{(1)} = \bigcup_{m=1,2,\cdots,N} W_m^{(1)} = \bigcup_{m=1,2,\cdots,N} \{M(m,j) \mid j = 1,2,\cdots,N\}$$

and

$$W = W^{(2)} = \bigcup_{n=1,2,\cdots,N} W_n^{(2)} = \bigcup_{n=1,2,\cdots,N} \{M(i,n) \mid j = 1,2,\cdots,N\}.$$

It is easy to see that the subset $W_m^{(1)}$ in $W^{(1)}$ and the subset $W_n^{(2)}$ in $W^{(2)}$ correspond to the $m^{th}$ row and the $n^{th}$ column in $M$, respectively. The $m^{th}$ row and the $n^{th}$ column in $M$ must intersect at the element $M(m,n) = w_t$ where $t = (m-1) \times N + n$, which corresponds to that subsets $W_m^{(1)}$ and $W_n^{(2)}$ must have one and only one common element $w_{mn} = W_m^{(1)} \cap W_n^{(2)} = w_t$.

Conversely, each element in $M$, $M(m,n) = w_t$, is uniquely identified by its coordinate $(m,n)$, which corresponds to the common element of the unique combination of $W_m^{(1)} \cap W_n^{(2)}$.

Hence, Lemmas 5 and 6 hold.

One of the simplest ways to partition set $W = \{1,2,\ldots,N^2\}$ into 2 different subset groups which satisfy Lemmas 5 and 6 are.

**Partition 1:**

$$W = \bigcup_{m=1,2,\cdots,N} W_m^{(1)} = \bigcup_{m=1,2,\cdots,N} \{M(m,j) \mid j = 1,2,\cdots,N\}$$

**Partition 2:**

$$W = \bigcup_{n=1,2,\cdots,N} W_n^{(2)} = \bigcup_{n=1,2,\cdots,N} \{M(i,n) \mid j = 1,2,\cdots,N\}.$$  

It is easy to verify that Partitions 1 and 2 are just the redundant routing wavelength subsets of Level 2 and Level 1 RDWRONs. According to Lemma 5, $W_m^{(1)}$ and $W_n^{(2)}$ has one and only one common element $w = W_m^{(1)} \cap W_n^{(2)}$. Then $w$ is the only wavelength which can be used to route for path $P$.

**B. Routing Scheme for RCWRON**

The following notations are used in our discussion.

$S_1 (1 \leq S_1 \leq N)$: the source node address of an $N^2$-RCWRON.

$D_1 (1 \leq D_1 \leq N)$: the destination node address of an $N^2$-RCWRON.

$S_2 (1 \leq S_2 \leq N)$: the source node address of Level1 RDWRON.

$D_2 (1 \leq D_2 \leq N)$: the destination node address of Level1 RDWRON.

$D_M (1 \leq D_M \leq N^2)$: the destination node address in the sub-network composed of all Level1 RDWRONs.

$S_M (1 \leq S_M \leq N^2)$: the source node address in the sub-network composed of all Level2 RDWRONs.

$w (w \in W)$: the routing wavelength for the whole $N^2$-RCWRON for a given $S$ and $D$.

$w_1 (w_1 \in W)$: the minimum routing wavelength of Level1 RDWRON for a given $S_1$ and $D_1$.

$w_2 (w_2 \in W)$: the minimum routing wavelength of Level2 RDWRON for a given $S_2$ and $D_2$.

According to the structure of RCWRON, we can obtain following equations.

$$S_1 = \text{mod}(S - 1, N) + 1$$

$$S_2 = \text{mod}(S_M - 1, N) + 1$$

$$D_1 = \text{mod}(D - 1, N) + 1$$

$$D_2 = \text{mod}(D - 1, N) + 1$$

$$D_M = \left[\frac{S - 1}{N}\right] \times N + D_1$$

According to Propositions 3 and 6, and Lemmas 5 and 6, we can obtain following equation.

$$w = w_1 + (w_2 - 1) \times N$$

Based on Eqns. (1)-(5), we can derive the following equations.

$$S_1 = \text{mod}(S - 1, N) + 1$$

$$S_2 = \left[\frac{S - 1}{N}\right]$$

$$D_1 = \text{mod}(D - 1, N) + 1$$

$$D_2 = \left[\frac{D - 1}{N}\right]$$

$$w_1 = \text{mod}(w - 1, N) + 1$$

$$w_2 = \left[\frac{w - 1}{N}\right]$$

and

$$D_1 = f_D(N, S_1, w_1)$$

$$D_2 = f_D(N, S_2, w_2)$$

$$S_1 = f_S(N, D_1, w_1)$$

$$S_2 = f_S(N, D_2, w_2)$$

$$w_1 = f_w(N, S_1, D_1)$$

$$w_2 = f_w(N, S_2, D_2)$$

Then we have the following results for routing in $N^2$-RCWRON.

**Proposition 7.** For an $N^2$-RCWRON, given the source node address $S$ and routing wavelength $w$, the destination node address $D$ can be derived as
\[ D = D_2 + (D_1 - 1) \times N, \]

where
\[
\begin{align*}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \left\lfloor \frac{S - 1}{N} \right\rfloor \\
w_1 &= \text{mod}(w - 1, N) + 1 \\
w_2 &= \left\lfloor \frac{w - 1}{N} \right\rfloor \\
D_1 &= f_D(N, S_1, w_1) \\
D_2 &= f_D(N, S_2, w_2)
\end{align*}
\]

**Proposition 8.** For an \(N^2\)-RCWRON, given the source node address \(S\) and the routing wavelength \(w\), the destination node address \(D\) can be derived as
\[ S = S_1 + (S_2 - 1) \times N \]

where
\[
\begin{align*}
D_2 &= \text{mod}(D - 1, N) + 1 \\
D_1 &= \left\lfloor \frac{D - 1}{N} \right\rfloor \\
w_1 &= \text{mod}(w - 1, N) + 1 \\
w_2 &= \left\lfloor \frac{w - 1}{N} \right\rfloor \\
S_1 &= f_S(N, D_1, w_1) \\
S_2 &= f_S(N, D_2, w_2)
\end{align*}
\]

Based on the discussion in the previous subsection, the routing wavelength for \(N^2\)-RCWRON can be derived as follows.

**Proposition 9.** For an \(N^2\)-RCWRON, given the source node address \(S\) and the destination node address \(D\), the routing wavelength \(w\) can be derived as
\[ w = W_1 \cap W_2 = w_1 + (w_2 - 1) \times N \]

where
\[
\begin{align*}
w_1 &= f_w(N, S_1, D_1) \\
w_2 &= f_w(N, S_2, D_2) \\
W_1 &= \{w_1, w_1 + N, w_1 + 2N, \ldots, w_1 + (N - 2)N, w_1 + (N - 1)N\} \\
W_2 &= \{(w_2 - 1)N + 1, (w_2 - 1)N + 2, \ldots, (w_2 - 1)N + (N - 1)w_2\}
\end{align*}
\]

and
\[
\begin{align*}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \left\lfloor \frac{S - 1}{N} \right\rfloor \\
D_2 &= \text{mod}(D - 1, N) + 1 \\
D_1 &= \left\lfloor \frac{D - 1}{N} \right\rfloor
\end{align*}
\]

**C. One Routing Example**

Here we use \(4^2\)-RCWRON as an example to illustrate the routing scheme. Fig. 7 shows the structure of Level 1 \(4^2\)-RDWRON and Level 2 \(4^2\)-RDWRON. The structure of the \(4^2\)-RCWRON is shown in Fig. 8.

The routing wavelength assignment for Level 1 and Level 2 RDWRON are shown in Tab. 2 (see Section III.C) and Tab. 3, respectively. The routing wavelength assignment for the whole \(4^2\)-RCWRON is given in Tab. 4.
VI. CONCLUSION

In this paper, we proposed a new 2-D Recursive Wavelength Routed Optical Network (RCWRON) based on the WRON, one type of on-chip interconnection suitable for ONoC. We first introduced the 2-D RDWRON and its routing scheme. Then we showed how to construct 2-D RCWRON using 2-D RDWRONs. The routing scheme for 2-D RCWRON is derived based on the routing scheme for 2-D RDWRON.

The major advantage of the proposed 2-D RCWRON over the WRON is its fault-tolerance capability. The tradeoff is its relatively high construction cost. Our future work includes study of alternative network structures which balance between the construction cost and the fault-tolerance capability.

APPENDIX

Proof of Propositions 1-3:

All rules and definitions used here are same as those in [8].

1) Step I

Consider the topology structure of the WRON as shown in Fig. 9. Each switch is indicated by its coordinate (c,r) uniquely in the network as introduced in [8]. When the routing wavelength w assigned to the routing path is different to all the wavelengths preset in the WRON, we refer this situation as the ‘The N-WRON is irrelative to the wavelength w’. When the N-WRON is irrelative to the wavelength, the relationship between the address of the source node S and the address of the destination node address D can be derived as \( D = N - S + 1 \).

Note that the integration of any number of N-SCs is equal to one N-SC and the integration of N-SC to other network (WRON) is equal to that network (WRON).

3) Step III

Given any \( N^2 \)-RDWRON and a wavelength \( w \) assigned to a special routing path, we can easily transform the RDWRON to a WRON by the following way.

In the \( N^2 \)-RDWRON, there are \( N \) N-WRON and N-1 IC blocks. Assume that 

\[
k = \left\lfloor \frac{w - 1}{N} \right\rfloor, \quad w_0 = \text{mod}(w - 1, N) + 1.
\]

Then the wavelength \( w \) exists only in the \( (k+1)^{th} \) N-WRON, i.e., all other \( k \)-1 N-WRONs are irrelative to the wavelength \( w \).

For the \( i^{th} \) N-WRON, \( 0 < i < k \), in the \( N^2 \)-RDWRON, it can be integrated with the \( i^{th} \) IC to an N-SC; for the \( j^{th} \) N-WRON, \( k < j \leq N \), in the \( N^2 \)-RDWRON, it can be integrated with the \( (j-1)^{th} \) IC to an N-SC too. Then the \( N^2 \)-RDWRON is composed only with SCs and WRONs which can be treated as WRON merely. This process is shown in Fig. 12.
\[ W = \{ w, w+N, w+2N, \ldots, w+(N-2)N, w+(N-1)N \} \]

where

\[ w = f_w(N, S, D), \]

and \( f_w \) is the function defined in Eqn. (3).

Hence, Propositions 1-3 hold.

![Diagram](image-url)

Figure 12 Transform \( N^2 \)-RDWRON to \( N \)-WRON.

Hence, the routing scheme of \( N^2 \)-RDWRON is almost same as that of \( N \)-WRON. The derivations of the destination address and the source address for \( N^2 \)-RDWRON is same as those for \( N \)-WRON. In deriving the routing wavelength, we can treat the \( N^2 \)-RDWRON as \( N \) different \( N \)-WRON and calculate them separately.

We summarize the routing scheme of RDWRON as follows.

For an \( N^2 \)-RDWRON, given the source node address \( S \) and the routing wavelength \( w \), the destination node address \( D \) can be derived as following.

\[ D = f_D(N, S, w_0) \]

where

\[ w_0 = \text{mod}(w-1, N) + 1, \]

and \( f_D \) is the function defined in Eqn. (1).

For an \( N^2 \)-RDWRON, given the destination node address \( D \) and the routing wavelength \( w \), the source node address \( S \) can be derived as following.

\[ S = f_S(N, D, w_0) \]

where

\[ w_0 = \text{mod}(w-1, N) + 1, \]

and \( f_S \) is the function defined in Eqn. (2).

In an \( N^2 \)-RDWRON, a set of different routing wavelengths can be used in routing from one source node to one destination node. Denote the set of different wavelengths of the \( N^2 \)-RDWRON as \( W \), given the RDWRON size \( N \), the source node address \( S \) and the destination node address \( D \), \( W \) can be derived as:

REFERENCES


